IV. "On the Theory of Internal Resistance and Internal Friction in Fluids; and on the Theories of Sound and of Auscultation." By ROBERT MOON, Esq., M.A., late Fellow of Queen's College, Cambridge. Communicated by ARTHUR CAYLEY, Esq. Received April 3, 1858.

(Abstract.)

The author shows in the first instance, that when sound is propagated along a cylindrical tube filled with air, the compression which takes place in any element calls forth a resistance which diminishes the velocity of the particles in the element, at the same time that the dilatation which takes place in any element calls into play a force which will tend to increase the velocity of the particles in the element. He considers that the amount of the force thus called into play (whether it be accelerative of, or retarding the motion) in an element of given magnitude in a given indefinitely short interval, will depend solely on the amount of compression or dilatation developed in the element in the interval, and the state of density in the element at the time; and he is thus led to the conclusion, that to the ordinary equation for the transmission of sound through a column of air must be added a term of the form

$$\pm b^2\left(\frac{dy}{dx}\right)^{-1}\frac{d^2y}{dx\,dt},$$

where x denotes the distance from the origin of the element when the air is at rest, y the same distance at the time t when the air is in motion, b^2 a constant depending on the compressibility of air under given circumstances; so that the accurate equation of sound (variation of temperature being neglected) will stand

$$\frac{d^2y}{dt^2}=a^2\left(\frac{dy}{dx}\right)^{-2}\frac{d^2y}{dx^2}\pm b^2\left(\frac{dy}{dx}\right)^{-1}\frac{d^2y}{dx\,dt}, \quad . \quad . \quad . \quad (1)$$

in which equation the upper or lower sign of b^2 is to be taken according as the motion of the particles is in the direction in which x is measured positively, or the contrary.

On the same principles the author shows that, in the case of elastic fluids, the general equations of motion, when internal resistance is taken into account, must be written as follows:—

$$\left.\begin{aligned}\frac{1}{\rho}\frac{dp}{dx}&=\mathrm{X}-\frac{d(u)}{dt}\pm k^2\rho\frac{du}{dx}\\ \frac{1}{\rho}\frac{dp}{dy}&=\mathrm{Y}-\frac{d(v)}{dt}\pm k^2\rho\frac{dv}{dy}\\ \frac{1}{\rho}\frac{dp}{dz}&=\mathrm{Z}-\frac{d(w)}{dt}\pm k^2\rho\frac{dw}{dz}\end{aligned}\right\}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where ρ denotes the density; X, Y, Z the impressed forces acting on the element; u, v, w the resolved parts of the velocity parallel to the coordinate axes; $\frac{d(u)}{dt}$ the total differential coefficient of u with respect to t, &c.; and k^2 replaces the b^2 of the preceding case. The author considers that, for moderate ranges of density, the above equations accurately represent the whole internal resistance.

It is next shown, that when the fluid is inelastic, the same equations will represent the motion, provided that we obliterate ρ in the terms involving k^2.

The force of internal friction in an elastic fluid in which the whole motion takes place parallel to the axis of x, and in which the whole lateral variation of motion transverse to the axis of x occurs in a direction parallel to the axis of z, is then shown to be properly represented by $\pm n^2\rho\frac{du}{dz}$, where n^2 is a constant depending on the nature of the fluid; the sign of the term to be introduced into the equation of motion being determined by the consideration that friction must always be a retarding force. The author thence derives the conclusion, that in order to represent the effect of internal friction in the motion of an elastic fluid, we must add to the first of equations (2) a term of the form

$$\pm n^2\frac{u}{\mathrm{V}^2}\cdot\rho\psi,$$

where

$$\mathrm{V}^2=u^2+v^2+w^2,$$

and

$$\psi^2=\left(u\frac{d\mathrm{V}}{dy}-v\frac{d\mathrm{V}}{dx}\right)^2+\left(w\frac{d\mathrm{V}}{dx}-u\frac{d\mathrm{V}}{dz}\right)^2+\left(v\frac{d\mathrm{V}}{dz}-w\frac{d\mathrm{V}}{dy}\right)^2;$$

and similarly with regard to the other two equations. When the fluid is inelastic, the terms in the equations of motion depending upon friction will be identical with those in the preceding case if we obliterate from the latter ρ.

Reverting to the equation of sound, which (neglecting terms of the second order) may be put under the form

$$\frac{d^2y}{dt^2}=a^2\frac{d^2y}{dx^2}\pm 2ae\frac{d^2y}{dx\,dt},$$

the author next shows that if the initial disturbance consist of a condensation alone, it will be transmitted with the velocity $a(1-e)$ in the direction in which its particles are moving; and that if it consists of a rarefaction alone, it will be transmitted with the velocity $a(1+e)$ in the direction contrary to that in which its particles are moving. It is here shown also incidentally, that whether the resistance be taken into account or not, the particles of a wave of condensation must all move in the same direction, which will be the direction of transmission; and the particles of a wave of rarefaction will all move in the same direction, which will be contrary to that of transmission.

In confirmation of the conclusion that waves of rarefaction are transmitted more rapidly than waves of condensation, the author adduces the fact, that when explosions of gunpowder have taken place, the glass in windows has been observed to break outwards rather than inwards.

It is then suggested, that, as when sound is produced, a condensation and rarefaction of air usually occur in immediate succession, if both kinds of disturbance were capable of affecting the human ear, we should hear sounds double; and as we know practically that this is not the case, it is contended that only one kind of disturbance, *i. e.* either rarefaction alone, or else condensation alone, can stimulate the ear.

It is shown to be *à priori* probable, that if one of the two classes of aërial disturbance is suppressed by the ear, that one would be disturbance by condensation, inasmuch as waves of rarefaction being swifter, would better perform the duty entrusted to them: and it is pointed out that if the sensation of sound is produced by aërial rarefactions alone, a difficulty attending the received theory will be obviated, by reason of the velocity deduced upon that theory being too small.

The author considers, however, that the question, whether either and which of the kinds of aërial disturbance is suppressed, can only be satisfactorily determined by examination of the ear itself. He

accordingly endeavours to establish, by arguments derived from the structure of the ear, that aërial rarefactions are alone capable of stimulating that organ in man. These arguments are briefly as follows:—

1. The tympanal membrane being convex inwards, a condensation could only affect the air in the tympanal cavity by stretching the membrane, which would cause an expenditure of force; whereas a rarefaction would produce the effect by a simple flexure of the membrane.

2. The sense of hearing being certainly produced by the motion of the fluid in the labyrinth, which is a closed vessel filled with an incompressible fluid, the requisite motion could not be produced by a compression of the atmosphere.

3. The disposition of the muscles of the ear is such as is calculated to assist and regulate the impressions produced by rarefactions rather than those produced by condensations.

4. The existence of the Eustachian tube is indispensable to the action of the organ (when all its other parts are in a normal state), on the supposition that sound is occasioned by rarefaction; whereas its uses are not satisfactorily predicable on the contrary hypothesis.

The author observes, that if rarefactions alone produce sound, it follows that a simple contraction of the muscles of the ear will render sounds inaudible. It follows also, on the same hypothesis, that a more delicate exercise of the same muscles will render the organ minutely susceptible to the influence of certain sounds, to the exclusion of others. It is urged also, that, admitting the action of these muscles to be to a large extent involuntary, there can be no doubt that by practice a great degree of command may be acquired over them. The author conceives that we may in this way account for the facility acquired by many persons of reading and writing, and of carrying on intricate trains of thought, without being disturbed by, or being conscious of, the noises around them. He conceives also that the same mode of explanation may be applied to account for the power of appreciating and analysing the most complex harmonies possessed by persons having a fine musical ear; which the author considers to be as certainly the result of specific mental and muscular training, as the faculty of vocalization, or the art of playing on a keyed instrument.

The author concludes by observing, that the equations for the transmission of an undulation along a musical string require a similar correction to that introduced in the case of aërial vibrations. The discussion of this branch of the subject he reserves for a future opportunity.

May 6, 1858.

The LORD WROTTESLEY, President, in the Chair.

In accordance with the Statutes, the Secretary read the following list of Candidates recommended by the Council for Election into the Society:—

Thomas Graham Balfour, M.D.
Edward Mounier Boxer, Captain R.A.
Frederick Currey, Esq.
David Forbes, Esq.
Alfred Baring Garrod, M.D.
William Henry Harvey, M.D.
The Rev. Samuel Haughton.
Henry Hennessy, Esq.
David Livingstone, LL.D.
John Lubbock, Esq.
Henry Darwin Rogers, LL.D.
William Scovell Savory, Esq., M.B.
Warington Wilkinson Smyth, Esq.
Lieut.-Col. Andrew Scott Waugh, B.E.
Thomas Williams, M.D.

The following communications were read:—

I. "On the Influence of Heated Terrestrial Surfaces in disturbing the Atmosphere." By THOMAS HOPKINS, Esq. Communicated by WILLIAM FAIRBAIRN, Esq. Received April 13, 1858.

(Abstract.)

In this paper the author stated that the Hadleian theory of winds, which is now the one generally recognized, is not supported by the evidence of facts, but rests on assumptions founded on imaginary effects of the partial expansion of the atmospheric gases by heat. It is assumed in that theory, that when the tropical heat expands these gases, they rise and flow away laterally in the higher regions towards the poles, from which they return to the tropics in the lower regions. But it was contended by the writer of the paper, that such heating